UNIVERSIDADE TÉCNICA DE LISBOA
INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

EXERCISES IN MICROECONOMICS

2009/2010

References:

- Gibbons, R. (1992), *A Primer in Game Theory*, Harvester Wheatsheaf (G)
- Mas-Collel, A., M. Whinston, and J. Green (1995), *Microeconomic Theory*, Oxford University Press, New York (MWG)
- Varian, H. (1992), *Microeconomic Analysis*, Norton, New York (V)

PRODUCTION

(Lectures 1, 2, and 3)

Remark: (*) signals those exercises that I consider to be the most important

Exercise 1

Draw production sets that a) violate and b) satisfy each of the following properties:

i. No free lunch;

ii. Possibility of inaction;

iii. Free disposal;

iv. Nonincreasing returns to scale ($y \in Y \Rightarrow \alpha y \in Y$, for all $\alpha \leq 1$);

v. Irreversibility ($y \in Y$ and $y \neq 0 \Rightarrow -y \notin Y$);

vi. Additivity ($y, y' \in Y \Rightarrow y+y' \in Y$).

Exercise 2 (includes MWG, Ex. 5.B.2 and 5.B.3)

Let $f(\cdot)$ be the production fuction associated with a single-output technology and let Y be its production set. Show that:

i. Y statisfies constant returns to scale if and only if $f(\cdot)$ is homogeneous of degree 1;

ii. Y is convex if and only if $f(\cdot)$ is concave;

iii. Y convex rules out the existence of economies of scale.

Exercise 3 (MWG, Ex. 5.B.6)

Suppose there are three goods. Goods 1 and 2 are inputs and good 3, with amounts denoted by q, is the output. Output can be produced by two techniques that can be operated simultaneously or separately. The first (respectively, the second) technique is specified by $\Phi_1(q_1)$ (respectively, $\Phi_2(q_2)$), the minimal amount of input 1 (respectively, 2) sufficient to produce q_1 (respectively, q_2). The two functions Φ_i $(\cdot)$ are increasing and $\Phi_i(0)=0$, $i=1,2$.

a) Describe the production set associated with these two techniques assuming free disposal.
b) Give sufficient conditions on $\Phi_i(\cdot)$, $i=1,2$, for the production set to display additivity.
c) Suppose that the input prices are w_1 and w_2. Write the first-order necessary conditions for profit maximization and interpret. Under which conditions on $\Phi_i(\cdot)$, $i=1,2$, will these conditions be sufficient?
d) Show that if $\Phi_i(\cdot)$, $i=1,2$, are strictly concave, then a cost-minimizing plan cannot involve the simultaneous use of the two techniques. Draw isoquants in the space of input uses.

Exercise 4

Show that if the production function is homogeneous of degree 1, the marginal rate of substitution is independent of the scale of production.

Exercise 5 (*)

Suppose that the production function takes the form $f(x)= (b_1x_1^{a}+ b_2x_2^{a})^{1/a}$.

i. Show that when a=1, isoquants become linear;
ii. Show that as $a\rightarrow 0$, this function comes to represent the Cobb-ouglas production function $f(x)= x_1^{b1}x_2^{b2}$;
iii. Show that as $a\rightarrow -\infty$, this function has in the limit the Leontief production function $f(x)=\min\{x_1,x_2\}$;
iv. compute the marginal rate of substitution and the elasticity of substitution for $f(\cdot)$.

Exercise 6 (*)

Derive the profit function and the supply correspondence for the following production functions:

i. $f(x)= x_1+ x_2$;

ii. $f(x)= \min\{x_1, x_2\}$;

iii. $f(x)= x_1^a x_2^b$, for a, b>0.

Exercise 7

Let $f(x)=10x-x^2/2$.

1. Determine the factor demand function;
2. Find the profit function.

Exercise 8

Establish all the properties of the cost function.

Exercise 9 (*)

Derive the cost function and conditional factor demand functions of the technologies given by:

i. $f(x)= x_1+ x_2$;

ii. $f(x)= \min\{x_1, x_2\}$;

iii. $f(x)= (x_1^a+ x_2^a)^{1/a}$, for $a\leq 1$

Exercise 10

Let $f(x_1, x_2, x_3, x_4)=\min\{x_1, x_2\}+ \min\{x_3, x_4\}$ and let $g(x_1, x_2, x_3, x_4)=\min\{x_1+x_2, x_3+x_4\}$.

1. determine the cost functions and the conditional factor demands for both production functions;
2. what kind of returns to scale does each technology exhibit?

Exercise 11

V, Ex. 4.4, p. 63

Exercise 12

V, Ex. 4.6, p. 63

Exercise 13

V, Ex. 5.2, p. 77

Exercise 14

V, Ex. 5.4, p. 77

Exercise 15

V, Ex. 5.6, p. 78

Exercise 16

V, Ex. 5.16, p. 79

Exercise 17

V, Ex. 5.17, p. 80

Exercise 18

Company A produces a single output q from two inputs x_1 and x_2. The following table contains two monthly observations concerning A's technology:

w_1	w_2	x_1	x_2	p	q
3	1	50	50	5	50
2	2	65	40	5	50

Can you recover A's technology?

Exercise 19 (*)

Determine the production functions and the conditional factor demands for the following cost functions:

i. $c(w_1,w_2,y)=y(w_1+2\,w_2)$;

ii. $c(w_1,w_2,y)=yw_1^{a}w_2^{b}$;

iii. $c(w_1,w_2,y)=y\min\{2w_1,w_2\}$.